A Practical Manual of Methodologies for Sampling of Consignments

Answers

Links between the Parameters and Tolerance Level

Parameters	Value	Remarks
Acceptance number	0	NPPO decide
Level of detection	0.1 – 1%	NPPO decide
Confidence level	95%, (99%, 99.9%)	usually 95%
Efficacy of detection	100%	assume 100%
Tolerance level	0%	actual value depends on other parameters
Sample size	Calculated from other parameters	

Binomial Distribution

$$P(X=i)=\binom{n}{i}p^{i}(1-p)^{n-i}$$

The binomial distribution is the discrete probability distribution of the number of successes in a sequence of *n* independent yes/no experiments, each of which yields success with probability *p*.

For example, take a sample of *n* units in which *p* are infested units. The binomial distribution describes the probability of exactly *i* units drawn from the consignment. The probability of binomial distribution is defined as the left formula. The parentheses indicate combination function.

The sample size is determined based on the probability of any infestation units which are not included in the sample is 95% when a sample which is drawn from the consignment whose infestation level is *p* is drawn.

There are three parameters, sample size (n), level of detection (p), confidence level (β).

Sample size (*n*)

$$n=\frac{\log_{e}(1-\beta)}{\log_{e}(1-p)}$$

Level of detection (*p*)

$$p=1-(1-\beta)^{\frac{1}{n}}$$

Confidence level (β)

$$\beta=1-(1-p)^{n}$$

Poisson Distribution

$$P(X=i)=\frac{(np)^i e^{-np}}{i!}$$

The Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

There are three parameters, sample size (n), level of detection (p), confidence level (β).

Sample size (n)

$$n=-\frac{\log_e(1-\beta)}{p}$$

Level of detection (p)

$$p=-\frac{\log_e(1-\beta)}{n}$$

Confidence level (β)

$$\beta=1-e^{-np}$$

Hypergeometric Distribution

$$P(X=\ i)=\frac{\binom{A}{i}\binom{N-A}{n-i}}{\binom{N}{n}}$$

The hypergeometric distribution is a discrete probability distribution that describes the number of infested units in a number of n units draws from a finite population without replacement. The hypergeometric distribution should be used to determine the sample size when sampling is done without replacement and the population size is finite.

Take, for example, a consignment of N units in which A are infested units. The hypergeometric distribution describes the probability of exactly i units drawn from the consignment. The probability of hypergeometric distribution is defined as the left formula.

Sample size (n)

$$n\cong(N-\frac{Np-1}{2})(\beta^{\frac{1}{Np}})$$

$$p=\frac{A}{N}$$

Box 1.1 Calculations sample size (n) according to binomial distribution and Poisson distribution from the following conditions.

1. Confidence level (β) 95%, level of detection (p) 0.1%, 0.5%, 1.0%.
2. Confidence level (β) 99%, level of detection (p) 0.1%, 0.5%, 1.0%.

Computation and Exercise

(binomial distribution)

β=0.95, p=0.001

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.95)}{\log_e(1-0.001)} = \frac{-2.9957}{-0.001} = 2995.7$$

(EXCEL)

$$n = LN(1-0.95)/LN(1-0.001) = 2994.23$$

β=0.95, p=0.005

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.95)}{\log_e(1-0.005)} = \frac{-2.9957}{-0.005} = 599.1$$

$$n = LN(1-0.95)/LN(1-0.005) = 597.65$$

β=0.95, p=0.01

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.95)}{\log_e(1-0.01)} = \frac{-2.9957}{-0.0101} = 296.6$$

(EXCEL)

$$n = LN(1-0.95)/LN(1-0.005) = 298.07$$

β=0.99, p=0.001

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.99)}{\log_e(1-0.001)} = \frac{-4.6052}{-0.001} = 4605.2$$

(EXCEL)

$$n = LN(1-0.95)/LN(1-0.005) = 4602.87$$

β=0.99, p=0.005

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.99)}{\log_e(1-0.005)} = \frac{-4.6052}{-0.005} = 921.0$$

(EXCEL)

$$n = LN(1-0.95)/LN(1-0.005) = 918.73$$

β=0.99, p=0.01

$$n = \frac{\log_e(1-\beta)}{\log_e(1-p)} = \frac{\log_e(1-0.99)}{\log_e(1-0.01)} = \frac{-4.6052}{-0.0101} = 456.0$$

$$n = LN(1-0.95)/LN(1-0.01) = 458.21$$

(Poisson distribution)

β=0.95, p=0.001

$$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.95)}{0.001} = -\frac{-2.9957}{0.001} = 2995.7$$

(EXCEL)

$$n = -\{LN(1-0.95)/0.001)\} = 2995.73$$

β=0.95, p=0.005

$$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.95)}{0.005} = -\frac{-2.9957}{0.005} = 599.1$$

(EXCEL)

$$n = -\{LN(1-0.95)/0.005)\} = 599.15$$

β=0.95, p=0.01

$$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.95)}{0.01} = -\frac{-2.9957}{0.01} = 299.6$$

(EXCEL)

$$n = -\{LN(1-0.95)/0.01)\} = 299.57$$

β=0.99, p=0.001 $$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.99)}{0.001} = -\frac{-4.6052}{0.001} = 4605.2$$

(EXCEL) $$n = -\{LN(1-0.99)/0.001)\} = 4605.17$$

β=0.99, p=0.005 $$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.99)}{0.005} = -\frac{-4.6052}{0.005} = 921.0$$

(EXCEL) $$n = -\{LN(1-0.99)/0.005)\} = 921.03$$

β=0.99, p=0.01 $$n = -\frac{\log_e(1-\beta)}{p} = -\frac{\log_e(1-0.99)}{0.01} = -\frac{-4.6052}{0.01} = 460.5$$

(EXCEL) $$n = -\{LN(1-0.99)/0.01)\} = 460.52$$

Box 1.2 Calculations level of detection (p) according to binomial distribution and Poisson distribution from the following conditions.

1. Confidence level (β) 95%, sample size (n) 300, 800 units.
2. Confidence level (β) 99%, sample size (n) 300, 800 units.

Computation and Exercise

(binomial distribution)

β=0.95, n=300 $$p = 1-(1-\beta)^{\frac{1}{n}} = 1-(1-0.95)^{\frac{1}{300}} = 0.00994$$

(EXCEL) $$p = 1-(1-0.95)\text{^}(1/300) = 0.009936$$

β=0.95, n=800 $$p = 1-(1-\beta)^{\frac{1}{n}} = 1-(1-0.95)^{\frac{1}{800}} = 0.003738$$

(EXCEL) $$p = 1-(1-0.95)\text{^}(1/800) = 0.037377$$

β=0.99, n=300 $$p = 1-(1-\beta)^{\frac{1}{n}} = 1-(1-0.99)^{\frac{1}{300}} = 0.015233$$

(EXCEL) $$p = 1-(1-0.99)\text{^}(1/300) = 0.015233$$

β=0.99, n=800 $$p = 1-(1-\beta)^{\frac{1}{n}} = 1-(1-0.99)^{\frac{1}{800}} = 0.005738$$

(EXCEL) $$p = 1-(1-0.99)\text{^}(1/800) = 0.005738$$

(Poisson distribution)

β=0.95, n=300 $$p = -\frac{\log_e(1-\beta)}{n} = -\frac{\log_e(1-0.95)}{300} = -\frac{-2.9957}{300} = 0.001$$

(EXCEL) $$p = -\{LN(1-\beta)/n\} = -\{LN(1-0.95)/300\} = 0.00999$$

β=0.95, n=800 $$p = -\frac{\log_e(1-\beta)}{n} = -\frac{\log_e(1-0.95)}{800} = -\frac{-2.9957}{800} = 0.0037$$

(EXCEL) $$p = -\{LN(1-\beta)/n\} = -\{LN(1-0.95)/800\} = 0.00474$$

β=0.99, n=300 $$p = -\frac{\log_e(1-\beta)}{n} = -\frac{\log_e(1-0.99)}{300} = -\frac{-4.6052}{300} = 0.015$$

(EXCEL) $$p = -\{LN(1-\beta)/n\} = -\{LN(1-0.99)/300\} = 0.01535$$

β=0.99, n=800 $\quad p = -\dfrac{\log_e(1-\beta)}{n} = -\dfrac{\log_e(1-0.99)}{800} = -\dfrac{-4.6052}{800} = 0.0058$

(EXCEL) $\quad p = -\{LN(1-\beta)/n\} = -\{LN(1-0.99)/800\} = 0.00576$

Box 1.3 Calculations <u>confidence level</u> (β) according to <u>binomial</u> distribution and <u>Poisson</u> distribution from the following conditions.

1. Level of detection (p) 0.01%, sample size (n) 300, 800 units.
2. Level of detection (p) 0.003%, sample size (n) 300, 800 units.

Computation and Exercise

(binomial distribution)

p=0.01, n=300 $\quad \beta = 1-(1-p)^n = 1-(1-0.01)^{300} = 95.059$

(EXCEL) $\quad \beta = 1-(1-0.01)$^$(300) = 95.059$

p=0.01, n=800 $\quad \beta = 1-(1-p)^n = 1-(1-0.01)^{800} = 0.99968$

(EXCEL) $\quad \beta = 1-(1-0.01)$^$(1/800) = 0.99968$

p=0.003, n=300 $\quad \beta = 1-(1-p)^n = 1-(1-0.003)^{300} = 0.59398$

(EXCEL) $\quad \beta = 1-(1-0.003)$^$(300) = 0.59398$

p=0.003, n=800 $\quad \beta = 1-(1-\beta)^n = 1-(1-0.003)^{800} = 0.90961$

(EXCEL) $\quad \beta = 1-(1-0.003)$^$(800) = 0.90961$

(Poisson distribution)

p=0.01, n=300 $\quad \beta = 1-e^{-np} = 1-e^{-300\times 0.01} = 0.95021$

(EXCEL) $\quad \beta = 1-EXP(-300*0.01) = 0.95021$

p=0.01, n=800 $\quad \beta = 1-e^{-np} = 1-e^{-800\times 0.01} = 0.99966$

(EXCEL) $\quad \beta = 1-EXP(-800*0.01) = 0.99966$

p=0.003, n=300 $\quad \beta = 1-e^{-np} = 1-e^{-300\times 0.003} = 0.59343$

(EXCEL) $\quad \beta = 1-EXP(-300*0.003) = 0.59343$

p=0.003, n=800 $\quad \beta = 1-e^{-np} = 1-e^{-800\times 0.003} = 0.90928$

(EXCEL) $\quad \beta = 1-EXP(-800*0.003) = 0.90928$

EXERCISE 1

Calculate value of blanks in the below table.

Confidence level	Level of detection	Sample size	
		binomial	Poisson
95%	5.0%	58.4	59.9
95%	0.75%	397.9	399.4
95%	0.3%	997.1	998.6
99%	5.0%	89.8	59.9
99%	0.75%	611.7	614.0
99%	0.3%	1532.8	1535.1
0.9184	0.5%	500	-
0.9179	0.5%	-	500
0.0952	0.1%	100	-
0.0952	0.1%	-	100
95%	0.001995	1,500	-
95%	0.001997	-	1,500
99%	0.003065	1,500	-
99%	0.003070	-	1,500

Box 1.4 Calculations <u>sample size (*n*)</u> according to <u>hypergeometric</u> distribution from the following conditions.

1. Consignment size (*N*) 1000, confidence level (β) 95%, level of detection (*p*) 0.1%, 0.5%.
2. Consignment size (*N*) 1000, confidence level (β) 99%, level of detection (*p*) 0.1%, 0.5%.

Computation and Exercise

N=1000, *p*=0.05, β =95%
(EXCEL)

$$n \cong (N - \frac{Np-1}{2})(1-(1-\beta)^{\frac{1}{Np}})$$
$$= (1000 - \frac{1000 \times 0.05 - 1}{2})(1-(1-0.95)^{\frac{1}{1000 \times 0.05}}) = 56.73$$

N=1000, *p*=0.01, β =95%
(EXCEL)

$$n \cong (N - \frac{Np-1}{2})(1-(1-\beta)^{\frac{1}{Np}})$$
$$= (1000 - \frac{1000 \times 0.01 - 1}{2})(1-(1-0.95)^{\frac{1}{1000 \times 0.01}}) = 257.70$$

N=1000, *p*=0.05, β =99%
(EXCEL)

$$n \cong (N - \frac{Np-1}{2})(1-(1-\beta)^{\frac{1}{Np}})$$
$$= (1000 - \frac{1000 \times 0.05 - 1}{2})(1-(1-0.99)^{\frac{1}{1000 \times 0.05}}) = 85.83$$

N=1000, *p*=0.01, β =99%
(EXCEL)

$$n \cong (N - \frac{Np-1}{2})(1-(1-\beta)^{\frac{1}{Np}})$$
$$= (1000 - \frac{1000 \times 0.01 - 1}{2})(1-(1-0.99)^{\frac{1}{1000 \times 0.01}}) = 367.38$$

EXERCISE 2

1. Country A imports a consignment which consists of 100,000 apple fruits from country B. Country A wants to detect consignment whose infestation rate exceeds over 0.6% at 95% confidence level. How many apple fruit does country A have to inspect? Calculate sample size according to binomial, Poisson and hypergeometric distribution.

   Binomial:

   498

   Poisson:

   500

   Hypergeometric:

   497

2. Country A imports a consignment which consists of 5,000 apple fruits from country B. Country A wants to detect consignment whose infestation rate exceeds over 0.6% at 95% confidence level. How many apple fruit does country A have to inspect? Calculate sample size according to binomial, Poisson and hypergeometric distribution.

   Binomial:

   498

   Poisson:

   500

   Hypergeometric:

   474

3. Country A imports a consignment which consists of 10 tons apple fruits from country B. Country A wants to detect consignment whose infestation rate exceeds over 0.6% at 95% confidence level. What is the weight of apple fruits country A has import, where the weight of one apple fruit is 200g. Calculate sample size according to binomial, Poisson and hypergeometic distribution. And transfers the sample size to weight. How much weight Country A inspects?

   Binomial:

Country A inspects 498 apples. Country A inspects 100 kg.

Poisson:

Country A inspects 500 apples. Country A inspects 100 kg.

Hypergeometric:

Country A inspects 498 apples. Country A inspects 100 kg.

4. Country A imports a consignment which consists of 1,000 branches of cut rose flowers from country B. Country A wants to detect the consignment whose infestation rate exceeds over 0.2% at 95% confidence level. How many branches does country A have to inspect? Where one branch of rose cut flower consists of ten cut flowers? Calculate sample size according to hypergeometric distribution. How many branches Country A inspects?

   Country A inspects 1390 cut rose flowers. Country A inspects 139 branches.

   If you consider one branch is a unit, in this case, you have to draw 777 branches as a sample.

5. Country A imports a consignment which consists of 10,000 of pear fruits from country B. One of the most serious pear fruit diseases has occurred in country B. Country A can permit 5 infested fruits which are included in the consignment at 95% confidence level. How many pear fruits do country A have to draw from the consignment?

   Country A’s level of detection is 0.0005. We can get level of detection as follow; divide 10,000 by 5, get 0.0005.

   Sample size is 4507.

6. Country A want to export mango fruits to County B. Country B states we draw 900 fruits as a sample in import inspection. What is the level of detection of Country B? Many counties set 95% confidence level.

   We can estimate level of detection is 0.33%.